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A CRITICAL REVIEW OF THE THEORY OF THE MERCURY STRAIN-GAUGE PLETHYSMOGRAPH*

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Abstract—A short introductory treatment of the basic theory of the mercury strain-gauge plethysmograph is presented. Some physiological aspects of this type of plethysmography are then discussed which, among other things, illuminate the difficulty in discriminating between skin-flow and muscular flow. A detailed calculation of the elastic influences follows and shows that one cannot generally expect a cancellation of these influences when comparing measurements with the calibration. This part illuminates further the difficulties in discriminating between skin-flow and muscular flow. As a result, such discrimination should be regarded as dubious. Some other sources of error are also discussed briefly. Two proposals for a new design are given. Finally, the Appendix shows the values of the gauge stretch which will give cancellation of the elastic errors in a simple model. The actual values cannot be used in practice but they indicate that the amount of stretch is quite critical.

List of symbols

a cross-section area of mercury column in gauge
A cross-section area of tissue cylinder (simplified model)
b side length of quadratic cross-section of gauge (simplified model)
d rigid part of gauge (due to mount)
E modulus of elasticity for tissue
E_g modulus of elasticity for gauge
k constant of proportionality
l length of mercury column in gauge
L length of tissue cylinder (simplified model)
n bone-to-tissue radius ratio (simplified model)
P arbitrary constant
Q arbitrary constant
r radius in circularly cylindrical tissue model
r_0 unrestrained radius of gauge turn
r_1 bone outer radius (circularly cylindrical model)
r_2 tissue outer radius (circularly cylindrical model)
r_{20} unrestrained value of r_2
R resistance of gauge
v volume of mercury in gauge
V volume of tissue cylinder (simplified model)
w_r radial shift of expanding tissue at initial radius r (simplified model)
$w_{r,2}$ w_r at $r = r_2$
δL length of tissue element
δV volume of tissue element
Δ increment of subsequent symbol
ϵ relative stretch of gauge
ν Poisson ratio for tissue
ρ resistivity of mercury
σ_r radial tension in tissue at radius r
Ω circumference of tissue at gauge

1. INTRODUCTION

THE PURPOSE of this report is twofold. The author first wishes to describe the theoretical background as a basis for an undergraduate thesis planned at the Division of Applied Electronics, Chalmers University of Technology. There seems, in the second place, to exist a certain overvaluation of the mercury strain-gauge method for plethysmography, emphasizing the need for a critical review.

Plethysmography is the study of variations in the blood filling of tissue. The study is commonly performed by venous occlusion plethysmography. In this method the venous return from a limb is prevented by an occluding cuff inflated to a supervenous but subdiastolic pressure. The resulting volume increase, which is linear until the distal venous pressure reaches the neighbourhood of the cuff pressure, is measured and recorded. The slope of this increase, divided by the basic tissue volume under study, gives the

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mean blood flow in the tissue. A more detailed description of plethysmography and its uses is given by SIGDELL (1968), together with a review of various methods.

One method for venous occlusion plethysmography is the mercury strain-gauge, originally described by WHITNEY (1953, 1954). It employs a mercury filled rubber tubing wound around a limb. Its resistance varies with its length (its stretch) and offers a simple method for recording variations in the circumference of the limb. The recording can be related to relative variations in volume. The method is convenient and simple to use but its reliability does not allow accurate measurements. Apparently many users are unaware of this fact. For many uses, however, an approximate value of the blood flow is quite sufficient, which can be determined in a convenient way with the mercury strain-gauge plethysmograph.

2. BASIC PRINCIPLES

Consider first a mercury column of length l, cross-section area a, resistivity ρ and volume v. If its resistance is R, then

$$\begin{cases} R = \rho \dfrac{l}{a}, \\ v = la, \end{cases} \tag{1}$$

which gives

$$R = \frac{\rho}{v} l^2. \tag{2}$$

Consider now a cylinder of length L, cross-section area A and cross-section circumference Ω. Elementary geometry states that

$$\Omega = k\sqrt{A}, \tag{3}$$

where k is a constant depending on the form of the cross-section. If a "mercury thread", i.e., a mercury channel so thin that the influence of its thickness at the bends can be neglected, of volume v and length Ω is wrapped around the cross-section area A, we have

$$R = \frac{\rho k^2}{v} A. \tag{4}$$

Denote the cylinder volume by V:

$$R = \frac{\rho k^2}{vL} V. \tag{5}$$

This is the basic relation for mercury strain gauge plethysmography.

In mercury strain-gauge plethysmography, however, only a short segment of the limb, of length δL and the volume δV, can be regarded as a cylinder. According to (5),

$$R = \frac{\rho k^2}{v\, \delta L} \delta V. \tag{6}$$

Volume variations in this segment can be studied through the measurement of resistance variations in the mercury strain-gauge:

$$\Delta R \propto \Delta(\delta V), \tag{7}$$

or better

$$\frac{\Delta R}{R} = \frac{\Delta(\delta V)}{\delta V}, \tag{8}$$

provided that the form of the cross-section area A does not change.

Variations in blood filling in the segment *follow* (8) *only if the expansion or contraction of the segment is purely radial*, i.e., if δL remains constant for the tissue-volume segment under study.

2. PHYSIOLOGICAL ASPECTS OF MERCURY STRAIN-GAUGE PLETHYSMOGRAPHY

One important comment is that the gradual filling of blood in the extremities (and other tissue) is influenced by gravity. If the limb is in a vertical position the increase will, therefore, start at the lowest part and gradually proceed upwards. Two strain gauges at different heights would give different recordings due to this effect. It is, on the other hand, to be expected that the cross-section area of a vertical limb changes conformally at those parts of the limb where the bones do not make up too large a part of this area.

If the limb is horizontal, the increase moves gradually upwards along the cross section area, so that not only its size but also its form is altered.

This provides one reason for serious doubts about this method for bloodflow measurement when high precision is required. Another reason is that the strain-gauge will be influenced by every source of venous return in tissues distal to the occluding cuff, and not only by the tissues in a neighbourhood of the cross section around which the gauge is wound. The discussion of a paper by WHITNEY (1954) at a Ciba Symposium, where some other physiological viewpoints are treated, is also of interest.

The mercury strain-gauge plethysmograph is convenient to use and, therefore, very useful for studies that do not require high precision, such as repeated comparisons and relative studies on one patient, comparisons between two patients during, for example, reactive hyperaemia, or an approximate value of the actual blood-flow. It is also useful as an aid in blood-pressure measurement [cf. SIGDELL (1968)].

For accurate absolute measurement of the mean blood flow in a certain tissue volume, the water-filled plethysmograph is the method of choice to-day [cf. SIGDELL (1968)]. Two important reasons are the following: with a suitable displacement transducer one can obtain a true and direct transmission of volume variations (which is not the case with any other method*) through fluid displacement; the calibration can be performed in a very exact way by injecting a known volume into the system (which automatically cancels several "sources of error", such as elastic influences at the seal etc.). Comparisons between the two methods are reported by WHITNEY (1953) and BURGER *et al.* (1959) and show a possible difference of 50 per cent in the worst case.

Some users of the mercury strain-gauge plethysmograph assume that they can discriminate between muscular flow and skin flow through measurements on different parts of the limb. They assume that the reading from a gauge around the proximal part of the forearm is mainly influenced by the muscular flow due to the large portion of muscular tissue in the cross-section of the arm at this level. They further assume that the reading from a gauge around the wrist is mainly influenced by the skin flow due to the lack of muscular tissue in the cross-section. Both these assumptions are tempting but the influence of sources of venous return in other parts of the limb, mentioned above, makes the assumptions doubtful. Considering the conditions for the formula (8), discrimination between the flows measured at the wrist and at another part of the arm (whatever their sources are) with the mercury strain-gauge method is also dubious.

The geometry and distribution of bone in the cross section at the wrist cause a variation in the form of the cross-section that is quite different from the change at the muscular part of the forearm. Furthermore the influence of gravity, mentioned above, may also cause discrepancies between the two readings. (See also the discussion following (25) in Section 3.)

3. TECHNICAL ASPECTS OF MERCURY STRAIN-GAUGE PLETHYSMOGRAPHY

Several sources of error result in a deviation from the ideal relation (8). Some are discussed by WHITNEY (1953) and BRAKKEE *et al.* (1966).† The mechanical sources will be discussed here in a detailed manner in the interest of clarity. This discussion will show that the common mechanical calibration procedure does not really eliminate the mechanical sources of error, contrary to a statement by BRAKKEE *et al.* (1966) and to the discussion by WHITNEY (1953). Furthermore the discussion by WHITNEY (1953) starts with an incorrect formula. Formula (1) of Appendix II in his paper has an incorrect sign in the denominator (which may be due to confusion between the angular and radial components of the tension at the limb surface). Furthermore, the

* The air-filled plethysmograph is analogous to the water-filled one but has larger sealing problems, has a higher temperature dependence, and is sometimes disturbed by pressure changes in the surrounding air.

† A correction to BRAKKEE *et al.* (1966): the formula following (2) in their paper should read $r_m = kl^2$ instead of $r_m = kl$ [cf. (2) above] which gives a cancellation of the errors mentioned there if $r_w = kgl$.

influence of the mount is not considered in the calculation by WHITNEY (1953).

The following formulae are valid for a stationary disc with constant thickness and rotational symmetry in its elastic properties [KÄRRHOLM (1957)]:
radial shift at radius r:

$$w_r = Pr + \frac{Q}{r}, \tag{9}$$

radial tension components at radius r:

$$\sigma_r = \frac{PE}{1-\nu} - \frac{1}{r^2}\frac{QE}{1+\nu}, \tag{10}$$

where ν is the Poisson ratio for the material and E its modulus of elasticity. P and Q are arbitrary constants determined by boundary conditions.

We apply these formulae to a circular disc of tissue of radius r_2 and with a concentric bone of radius r_1. Therefore $w_r(r_1) = 0$, or

$$Q = -Pr_1^2. \tag{11}$$

Now suppose the disc to be surrounded by a thin strain-gauge of quadratic cross section $b \times b$ ($b \ll r_2$) and of relaxed radius r_0. We also suppose that a portion d of the gauge is rigid due to the mount. When the gauge is distended to the radius r_2 its radial tension at the inner surface will be

$$\sigma_r(r_2) = \frac{b}{r_2} E_g \frac{r_2 - r_0}{r_0 - d/2\pi} \tag{12}$$

where E_g is the modulus of elasticity for the gauge.

Equations (12) and (10) determine the value of P [$r = r_2$ in (10)] and we obtain for $w_r(r_2) = w_{r,2}$:

$$w_{r,2} = \frac{bE_g(1-\nu^2)}{E(r_0 - d/2\pi)} \frac{(r_2 - r_0)(r_2^2 - r_1^2)}{(1+\nu)r_2^2 + (1-\nu)r_1^2}. \tag{13}$$

The relaxed radius of the tissue is

$$r_{20} = r_2 + w_{r,2} \tag{14}$$

and if r_2 is increased by Δr_2 the gauge is extended by $2\pi \Delta r_2$ corresponding to

$$\Delta r_{20} = \Delta r_2 + w_r(r_2 + \Delta r_2) - w_r(r_2), \tag{15}$$

or

$$\Delta r_{20} \approx \Delta r_2 \left(1 + \frac{\partial w_{r,2}}{\partial r_2}\right), \tag{16}$$

where

$$\frac{\partial w_{r,2}}{\partial r_2} = 1\cdot 5 \frac{bE_g}{E(r_0 - d/2\pi)} \frac{3r_2^4 + 6r_1^2 r_2^2 - r_1^4 - 8r_1^2 r_2 r_0}{(3r_2^2 + r_1^2)^2} \tag{17}$$

if $\nu = 0\cdot 5$, which can be assumed for the tissue [cf. WHITNEY (1953)].

Now suppose that d is altered by the quantity Δd, for the purpose of calibration. This alters the length of the strain-gauge by $\Delta d - 2\pi \Delta w_{r,2}$. We have ($\Delta r_2 = -\Delta w_{r,2}$)*:

$$w_{r,2} + \Delta w_{r,2} \approx w_{r,2} - \Delta w_{r,2} \frac{\partial w_{r,2}^*}{\partial r_2} + \Delta d \frac{\partial w_{r,2}}{\partial d}, \tag{18}$$

from which

$$\Delta w_{r,2}\left(1 + \frac{\partial w_{r,2}}{\partial r_2}\right) = \Delta d \frac{\partial w_{r,2}}{\partial d}. \tag{19}$$

Here

$$\frac{\partial w_{r,2}}{\partial d} = \frac{w_{r,2}}{2(\pi r_0 - d/2\pi)} \tag{20}$$

and the alteration of the gauge length becomes

$$\Delta d \frac{1 + \frac{\partial w_{r,2}}{\partial r_2} - \frac{w_{r,2}}{r_0 - d/2\pi}}{1 + \frac{\partial w_{r,2}}{\partial r_2}}. \tag{21}$$

Here

$$\frac{\frac{w_{r,2}}{r_0 - d/2\pi}}{\frac{\partial w_{r,2}}{\partial r_2}} = \frac{\epsilon r_2}{(r_0 - d/2\pi)\left(1 - \frac{4\epsilon}{1\cdot 5 - n^2 - 0\cdot 5n^4}\right)}, \tag{22}$$

* $r_2 + w_{r,2} = r_{20}$ = constant in this case.

where

$$\epsilon = \frac{r_2 - r_0}{r_2} \tag{23}$$

and

$$n = \frac{r_1}{r_2}. \tag{24}$$

If the relative stretch ϵ of the gauge (including the mount) is sufficiently small,

$$\frac{w_{r,2}}{r_0 - d/2\pi} \approx \epsilon \frac{r_2}{r_0} \frac{1}{1 - d/2\pi r_0} \frac{\partial w_{r,2}}{\partial r_2} \ll \frac{\partial w_{r,2}}{\partial r_2}. \tag{25}$$

The alteration of the gauge length in this case is therefore approximately Δd. *This shows that the influence of the elasticities is much less at calibration than at measurement if the stretch is small. Cancellation can occur at a large stretch, but the value of the stretch is then quite critical*: see Appendix.

The discussion has assumed a gauge of square cross-section to simplify the calculation. There is no reason to expect a much different result with a circular cross-section. Equations (16) and (17) show that the difference between the restrained and relaxed change of radius of the tissue may not be negligible. The resistance change is proportional to the restrained change of radius.

The discussion also assumes a circular cross-section of the limb with a circular and concentric bone. It furthermore neglects the elastic influence from the surrounding tissue. Therefore (17) can hardly be used for estimation when measuring on the wrist, for example. One may, however, expect a much smaller deviation from (17) for the muscular portion of the forearm or other 'fleshy' parts of the limbs. *This consideration illuminates further the difficulties in discriminating between muscular flow and skin flow mentioned in the last part of Section* 2.

It should also be pointed out that the mount is another cause of non-linearity. This is clear from (2) and (6) rewritten for the case where the mount causes a part of the gauge with length d to be rigid.

$$R = \frac{\rho}{v} (\Omega - d)^2 \tag{26}$$

and

$$R = \frac{\rho k^2}{v\, \delta L} \left[\delta V + \frac{d^2 \delta L}{k^2} - \frac{2d}{k} \sqrt{(\delta V \delta L)} \right]. \tag{27}$$

Normally, however, the variations are so small that they can be considered linear. This has also been assumed in the discussion above.

Another source of error influences the measurement if the calibration is carried out as by Brakkee *et al.* (1966). The wiring resistance will, in this case, influence the result. Their calibration method will not, however, be considered here, since its practical advantages are quite small.

Thermal effects will also influence the result in the form of a resistance drift. If d.c. measurement of resistance is used, thermoelectrical potentials will add another drift component. There is reason to expect that these drifts are so slow that they will have a negligible effect on a plethysmogram.

Gravitation can be assumed to have some effect on the strain gauge itself, as mercury is quite heavy, if it is mounted around a vertical cross section. Furthermore, one must make sure that no part of the gauge is pressed between the limb and a support.

Finally, the ends of the strain gauge are often lifted somewhat from the surface of the limb at the mount, reducing the contour of contact between gauge and limb with the most common type of mount. This effect is another source of error, which has not been considered in the calculation above. The effect should be reduced by an improved design.

4. SHORT PROPOSALS FOR A NEW DESIGN

To avoid the lifting of the gauge at the mount, mentioned in the last part of Section 3, and simplify calibration, a new type of mount is proposed, sketched in Fig. 1. The strain gauge is fixed in a short part of a helical groove with a circular cross-section, much like a jam cleat (used on sailing-boats). The helical form of the groove allows the gauge to come out of it at limb surface level. The triangular cross-section

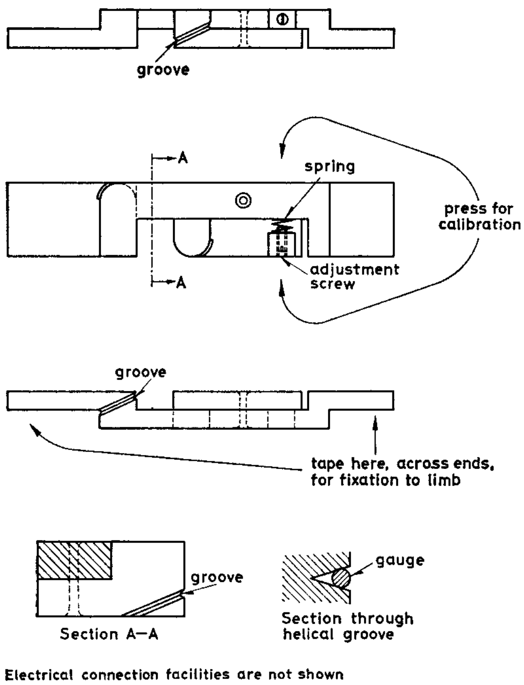

FIG. 1. Proposed mount for the mercury strain-gauge.

of the groove pinches the gauge without much compression (the compression is, in any case, constant except for a very short and negligible part at the lower end of the groove). One can easily use two or more turns of gauge. Calibration is effected by pressing together the two pieces of the mount at one end so that the fixations for the gauge move, relatively to each other, by a certain distance which can be set with the adjustment screw. The mount should be fixed to the skin with adhesive tape. There may be an effect due to reduction of friction between skin and gauge by means of a suitable skin cream; whether this is so should be investigated experimentally.

A simple and suitable method for the measurement of the resistance is proposed in Fig. 2. To avoid influence of the lead resistance, voltage is sensed at the gauge by special leads. This lead resistance does not influence the recorded resistance variation, either during plethysmography or during calibration, but a different setting of the reset knob would be required for different lead lengths if the voltage were sensed

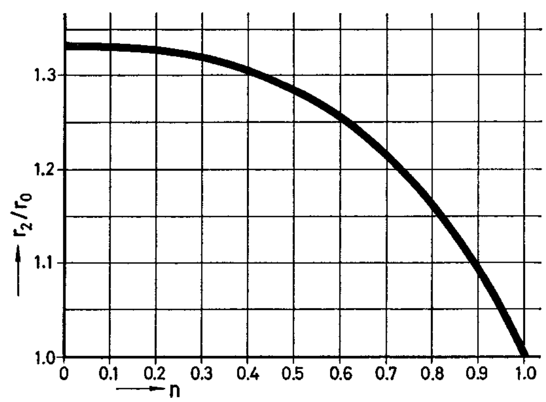

FIG. 3

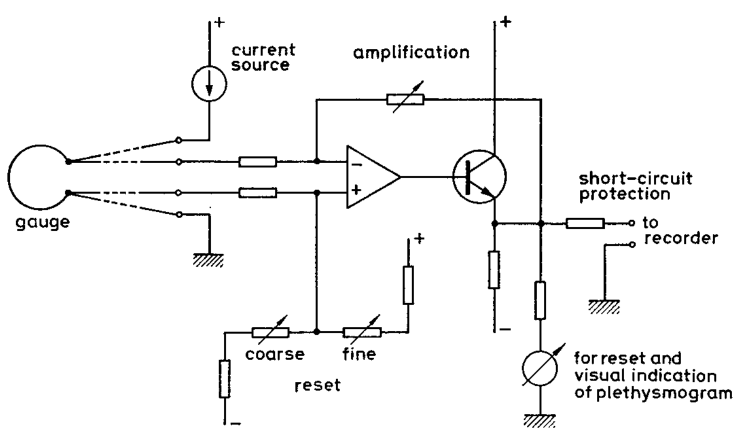

FIG. 2. Proposed method for the measurement of the resistance variations.

at the far end (from the gauge) of the current lead.

The detailed design was carried out by students of Chalmers University of Technology and has been published as an undergraduate thesis. (SCHAFFER and STRÖMBERG, 1968).

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APPENDIX. STRETCH REQUIRED FOR CANCELLATION OF THE ELASTIC INFLUENCES AT CALIBRATION AND MEASUREMENT WHEN $d \approx 0$ ($d \ll r_0$)

Cancellation occurs when the coefficient of $\partial w_{r,2}/\partial r_2$ in (22) is unity. Figure 3 gives the ratio r_2/r_0 required for cancellation as a function of the bone-to-tissue radius ratio n. The value is quite critical: for example $r_2/r_0 = 1{\cdot}33$ instead of $1{\cdot}28$ at $n = 0{\cdot}5$ gives an influence from the elasticities which is almost twice as great at calibration as at measurement. The values in the diagram are calculated for a simplified model (Section 3) and may be quite different, but not less critical, in practice.

UNE REVUE CRITIQUE DE LA THÉORIE DES JAUGES DE CONTRAINTE AU MERCURE APPLIQUÉES À LA PLÉTHYSMOGRAPHIE

Sommaire—L'auteur commence par un rapide traité sur la théorie des jauges de contrainte au mercure utilisées en pléthysmographie. Certains aspects physiologiques de ce type de pléthysmographie sont présentés et mettent en évidence, entre autre, la difficulté de distinguer le flux cutané de flux musculaire. Un calcul détaillé des influences élastiques montre qu'il ne faut pas s'attendre à annuler ces influences par comparaison des mesures aux valeurs d'étalonnage. Ceci conduit vers d'autres difficultés dans la discrimination entre flux cutané et flux musculaire. En fait une telle discrimination doit être considerée comme douteuse. L'auteur signale brièvement d'autres sources d'erreur et présente deux suggestions en vue d'une autre méthode, Enfin, en appendice, on trouvera les valeurs des tensions de la jauge qui permettent, pour un modèle simple, d'obtenir une annulation des erreurs élastiques. Les valeurs réelles ne peuvent être utilisées en pratique mais elles montrent que cette tension est critique.

KRITISCHER ÜBERBLICK ÜBER DIE THEORIE DER QUECKSILBER-DEHNUNGSMESSUNGS-PLETHYSMOGRAPHEN

Zusammenfassung—Es wird eine kurze Einführung der grundlegenden Theorie der Plethysmographie mit Quecksilber-Dehnungsmeßstreifen gegeben. Dann werden einige physiologische Aspekte dieses Plethysmographieverfahrens diskutiert. Unter anderem wird die Schwierigkeit hervorgehoben, zwischen Haut- und Muskeldurchfluß zu unterschieden. Es folgt eine ausführliche Berechnung der elastischen Einflüsse. Diese zeigt, daß man im allgemeinen diese Einflüsse nicht vernachlüssigen kann, wenn die Messungen mit den Eichwerten verglichen werden. In diesem Abschnitt werden weitere Schwieriegkeiten der Unterscheidung zwischen Haut- und Muskeldurchfluß aufgezeigt. Solche Unterscheidungen erweisen sich somit als zweifelhaft. Einige weitere Fehlerquellen werden kurz diskutiert. Zwei Vorschläge für Neukonstruktionen werden gegeben. Schließlich sind in einem Anhang Dehnungswerte aufgeführt, welche in einem einfachen Modell die elastischen Fehlerquellen verschwinden lassen. Die Werte können in der Praxis nicht verwendet werden, sie zeigen jedoch, daß der Betrag der Dehnung recht bedeutungsvoll ist.

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